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GASTROINTESTINAL TRANSIT TIME:- AN IN VITRO ANALYSIS OF THE VARIOUS PARAMETERS CONTROLLING THE CRITICAL FLOW VELOCITY OF PARTICLES IN A HORIZONTAL TUBE

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Abstract:

The factors which affect the critical flow velocity (V_C) of particles of barium sulphate, bismuth subcarbonate, kaolin, sulphadiazine and latex particles has been determined in a horizontal tube. These factors were particle size, particle density, fluid viscosity and tube diameter. V_C was found to increase with increasing particle diameter, particle density and tube diameter but decreases as viscosity of the flowing fluid . The results obtained were found to fit the models of Wicks, Durand and Wasp for the flow conditions of settled beds .

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Introduction

When a powder is injected into a tube through which a fluid is flowing, one of the following situations prevails: (1,2). (See Fig. 1).

a) The particles migrate to the tube wall and remain stationary.

b) The particles migrate to the tube wall and slide or roll along the tube. This tybe of flow is referred to as sliding bed.

c) The particles flow at approximately the same rate as the fluid. This is referred to as heterogeneous flow.

d) The particles remain suspended and flow at the same rate as the fluid. This is referred to as homogeneous flow.

Situation 'd' is rarely encountered in practice, but situation 'b' and 'c' are common. Each system occurs at a given flow rate of the fluid. The flow rate at which movement of the particles occurs is referred to as the critical velocity. In this study, type 'b' only is investigated. Other factors which also determine the type of flow regime occurring include bouyancy, lift and drag. Where as the bouyancy effect is constant for a given powder, the drag and lift effects vary with the flow conditions. For example, at low flow rates drag and lift effects are too small to overcome the bouyancy effect. Therefore, the particles migrate to the tube wall and remain there. At moderately high flow rates, the drag and lift effects become significant and the particles become suspended in the fluid, and maintained by turbulence. At relatively high speed, the particles become fully suspended and homogeneous flow ensues.

A similar situation may exist when a drug is given orally in a solid dosage form. After disintegrating in the stomach, the particles enter the intestine, the rate at which they travel along the intestine depends on factors such as the intestinal mobility, the particle diameter, shape, density and viscosity of the fluid flowing through the intestine. In a

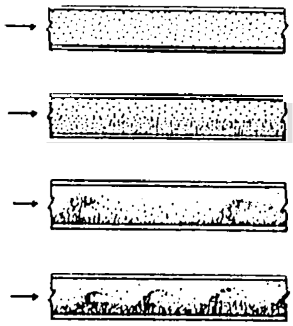

Fig. 1 Representative plot of slurry flow modes

previous work (3), the effect of these factors on the mean relative residence time of particles flowing in a horizontal tube was investigated. The purpose of this work is to study how the above factors influence the critical flow velocity (Vc) of particles moving in a horizontal tube. The usefullness and limitations to this model as a representative of a horizontal segment of the intestine have been discussed elsewhere (3).

Theory:

Critical velocity of slurries, Vc, is defined as the velocity below which a stationary deposit of solids forms in the pipe, thus it represents the lowest velocity at which the system can be operated such that stable flow conditions prevail. For estimating Vc, Durand (4) presented the empirical equation:

$$Vc = F_L\sqrt{2gD(s-1)} \qquad \text{....(1)}$$

in which F_L is given graphically as a function of particle diameter and solids concentration. Durand found that with particle above 1mm, F_L is essentially independent of particle size and solids concentration. Equation (1) can be rearranged into the following form:

$$F_L = \frac{V_c}{\sqrt{2gD}} \cdot \frac{1}{\sqrt{S-1}} \qquad \text{....(2)}$$

The term F_L is in fact a modified Froude Number at deposition. Spells (5) correlated data from the literature for Vc with a combination of Froude and Reynolds numbers. Spells's correlation has the form:

$$Vc^{1.225} = 0.0251\ gd_p\ (S-1)\ \left(\frac{D\rho}{\mu}\right)^{.775} \qquad ....(3)$$

Hughmark (6), Gregory (7), Thomas (8), wasp et al (9) and Zandi (10) developed correlations of the same nature. Using data on fine and coarse sand slurries, Hayden and Stelson (11) found that the correlations of Durand, Hughmark and Zandi yielded approximately the same degree of agreement. Spells equation yielded poor results, but the particle sizes of the solids used in the experiments were larger than those used by spells.

Wasp et al (9) used data from Durand (4), Sinclair (12), Yotsukura (13) and Wicks (14) together with the correlation given by Eq. (1). Wasp et al (9) reasoned that an equation of the form

$$Vc = F_L'\ \sqrt{2gD(S-1)}\ \left(\frac{dp}{D}\right)^{1/6} \qquad ....(4)$$

may prove to be a possible correlation tool, in which F_L' is given graphically as a function of solids concentration only. Wicks (14) developed a mathematical model which interrelates various factors such as flow rate, pipe diameter, fluid viscosity and density difference between the particles and fluid. He obtained the following relationship:

$$\Psi = 0.1\ S^3 \qquad \text{for } S < 40 \qquad ....(5)$$

where the functions Ψ and S are defined as follows:

$$\Psi = \frac{\rho\ d_p\ V_c^4}{(\rho_p - \rho)\ g\mu^2} \qquad ....(6)$$

$$S = \frac{D_{eq}\ V_c\ \rho}{\mu}\ \left(\frac{d_p}{D}\right)^{2/3} \qquad ....(7)$$

When Eqs. (5), (6), and (7) are rearranged one can obtain an explicit expression for the critical velocity in the following form:

$$V_c = 0.1\ gD\ (s-1)\ \frac{\rho d}{\mu} \qquad \ldots.(8)$$

where:

ρ = fluid density, g cm^{-3}

ρ_p = particle or solid density, g cm^{-3}

s = ρ_p/ρ , dimensionless

d_p = particle diameter, cm

D = pipe diameter, cm

D_{eq} = equivalent diameter of the flow region above the bed, cm = $D-d_p$.

g = gravitational acceleration, cm sec^{-2}

μ = viscosity of the fluid, g cm^{-1} sec^{-1}

Vc = critical velocity, cm sec^{-1}.

For the purpose of comparison, three correlations similar to those of Durand, Wasp and Wicks, are presented in this work. In our experiments, since the bed depth was one particle thick, i.e. $dp \ll D$, the height of the bed was considered negligible compared to the tube diameter and hence D_{eq} is equivalent to D.

In addition, it must be noted that the experimental determination of V_c is subject to difference in technique, subjective judgement of the observer and to experimental error, and it can be seen from the dimensional analysis presented in the Appendices that the different approaches of Durand, Wasp and Wicks are supported by their experimental findings.

Materials and Methods

Materials:

The materials used are described elsewhere (3). Sulphadiazine was obtained from Sigma Chemical Co., St. Louis, U.S.A.

Methods:

Tygon tubing 50 cm in length was connected to a Masterflix peristalitic pump. Liquid was then pumped through the tube. The pump was stopped and 0.5 ml of a 10% suspension was injected. The pump was then restarted immediately after injection and the flow rate was adjusted till particles from different regions of the bed were seen to slide along the tube. The flowing liquid was then collected by means of a fraction collector and the critical velocity (Vc) was calculated.

Effect of particle Size on Vc:- 10% W/W suspensions of different size fractions of barium sulphate and bismuth subcarbonate were prepared. 0.5 ml% W/V of each suspension was injected into the tube and the flow rate was adjusted till the particles were seen to slide along the tube. The flow rate at this speed was determined from which Vc was calculated for every particle size fraction. A plot was then made for particle size versus Vc.

Effect of Particle Density on Vc:- 10% W/V suspensions of barium sulphate, bismuth subcarbonate, magnesium carbonate, talc and sulphadiazine particle size fraction 128 μm were prepared. 0.5 ml of each suspension was injected into the tube separately and Vc was calculated as previously described. A plot was then made of density against Vc.

Effect of Viscosity on Vc:- In order to study the effect of viscosity on Vc, a series of solutions containing different concentrations of methocel were prepared. The viscosities of these solutions were measured using an Ostwald viscometer. The viscosities were found to range from 0.01-0.08 cP. 10% W/V suspensions of barium sulphate and talc (particle size fraction 128 μm) were prepared in all solutions. 0.5 ml of each suspension was injected separately into the tube and Vc was calculated as previously described. A plot was then made of viscosity against Vc.

Effect of Tube Diameter on Vc:- In order to study the effect of tube diameter on Vc, five tubes were used with diameters ranging from 0.07-0.3 cm. 10% W/V suspensions of barium sulphate and talc of particle size 128 μm were prepared. 0.5 ml of each suspension was separately injected in each tube and Vc was determined as previously described. A plot was then made of tube diameter against Vc.

RESULTS AND DISCUSSION

Fig. 2 shows the effect of particle size on Vc. It can be seen that a linear relationship between particle size and Vc exists for both barium sulphate and bismuth subcarbonate. The figure also indicates that the increase in Vc with increasing particle size is not very pronounced. The graph also shows that for any one particle size, a greater Vc was obtained for bismuth subcarbonate than barium sulphate, thus signifying the effect of density. Similar trends were obtained when particles of the same size but different densities were studied (Fig. 3). The figure shows that a linear relationship between density and Vc exists. The higher the density, the greater was Vc. In contrast, increasing the viscosity appeared to decrease Vc for a given particle size (Fig. 4).

Fig. 5 illustrates the relationship between the tube diameter and Vc for barium sulphate and talc. The graph shows that for both barium sulphate and talc, Vc increased initially with tube diameter and then a plateau region was obtained indicating that increasing the tube diameter beyond a 0.22 cm results in a little change in Vc. Similar findings have been reported by Sinclair (12).

For the purpose of comparison with the previously published correlations three plots were made; Fig. (6) is a Wicks's plot which correlates the various variables discussed above together by plotting log ψ versus log S. The graph indicates that a nonlinear relationship

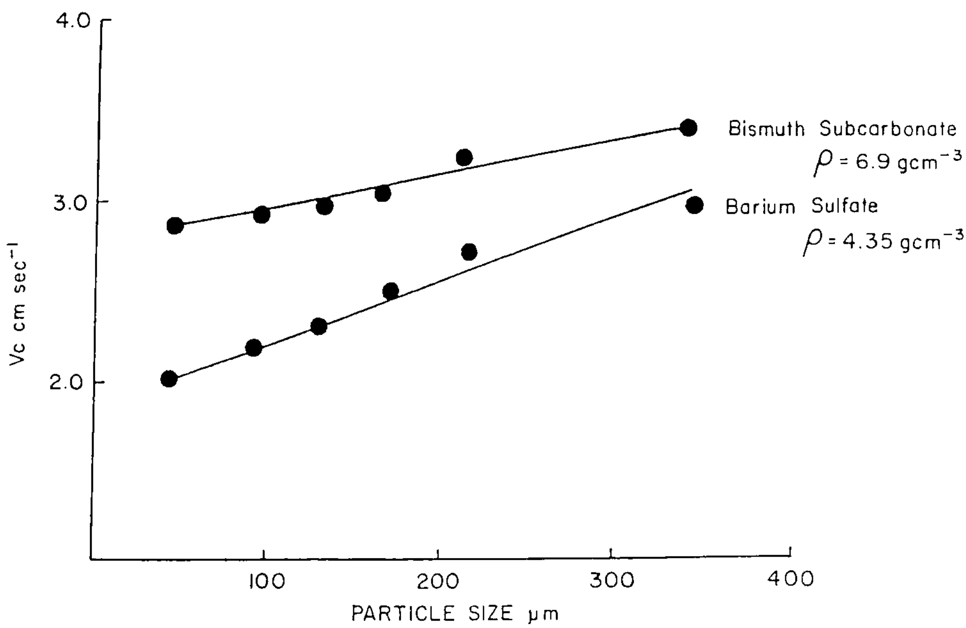

Fig. 2 The effect of particle size on the critical velocity (Vc).

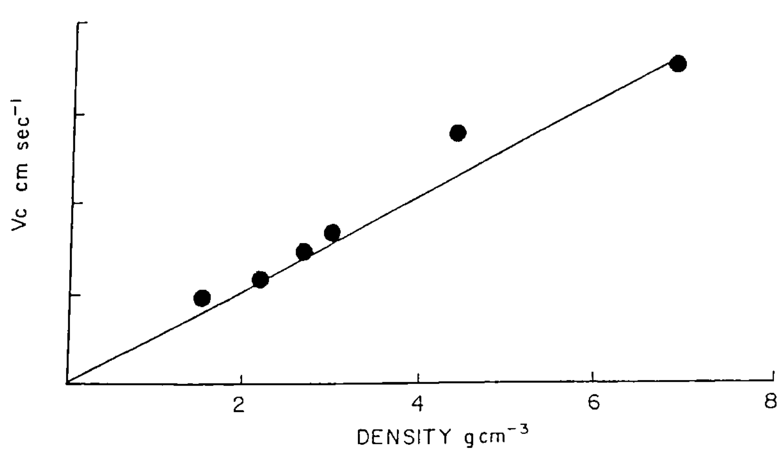

Fig. 3 The effect of particle density on the critical velocity (Vc).

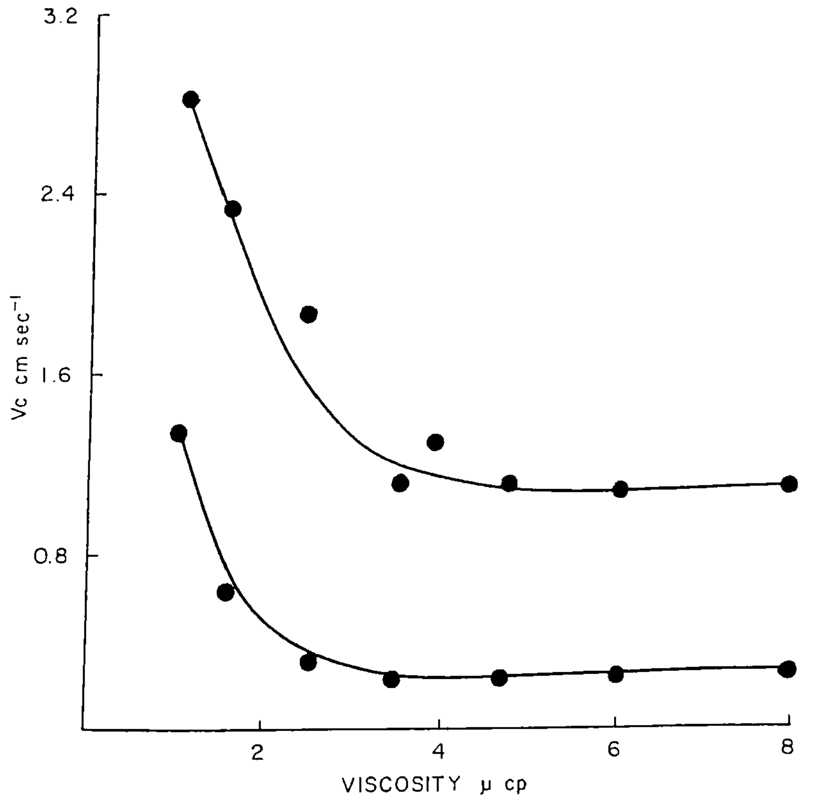

Fig. 4 The effect of Viscosity (μ) on the critical velocity (Vc).

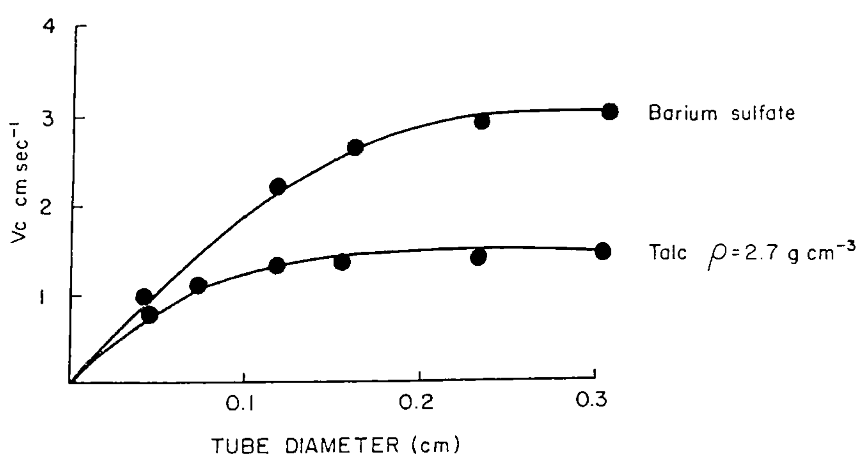

Fig. 5 The relationship between the tube diameter and the critical velocity (Vc).

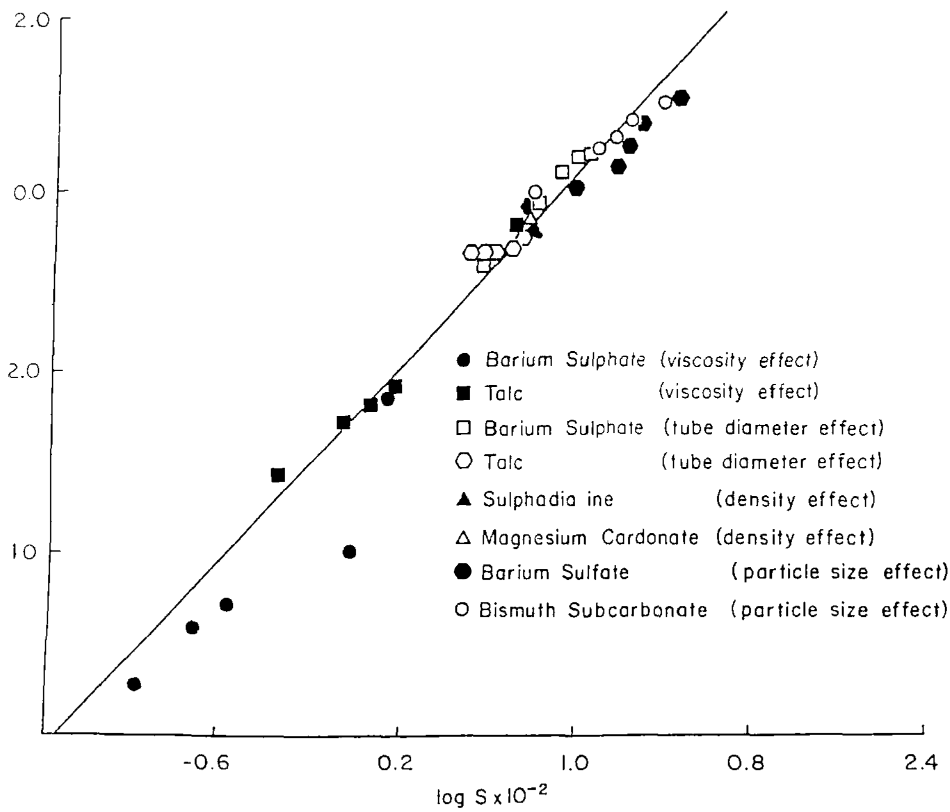

Fig. 6 The Wicks's plot, showing the relationship between log S and ψ.

between ψ and S exists as have been reported by Wicks. Figures (7) and (8) are Durand's and Wasp's plots respectively.

In order to simplify the process of comparison Table 1 is constructed to indicate the correlations obtained in the present work accompained with those reported by Wicks, Durand and Wasp.

It can be noted that the closet correlation that the present data fit is Wicks's (correlation coefficient is 0.95 number of points is 32). On the other hand, there exists a good agrement between the data obtained in this work and those of Durand and Wasp. However, Wicks's correlation is more relevant to represent our data than those of Durand and Wasp since

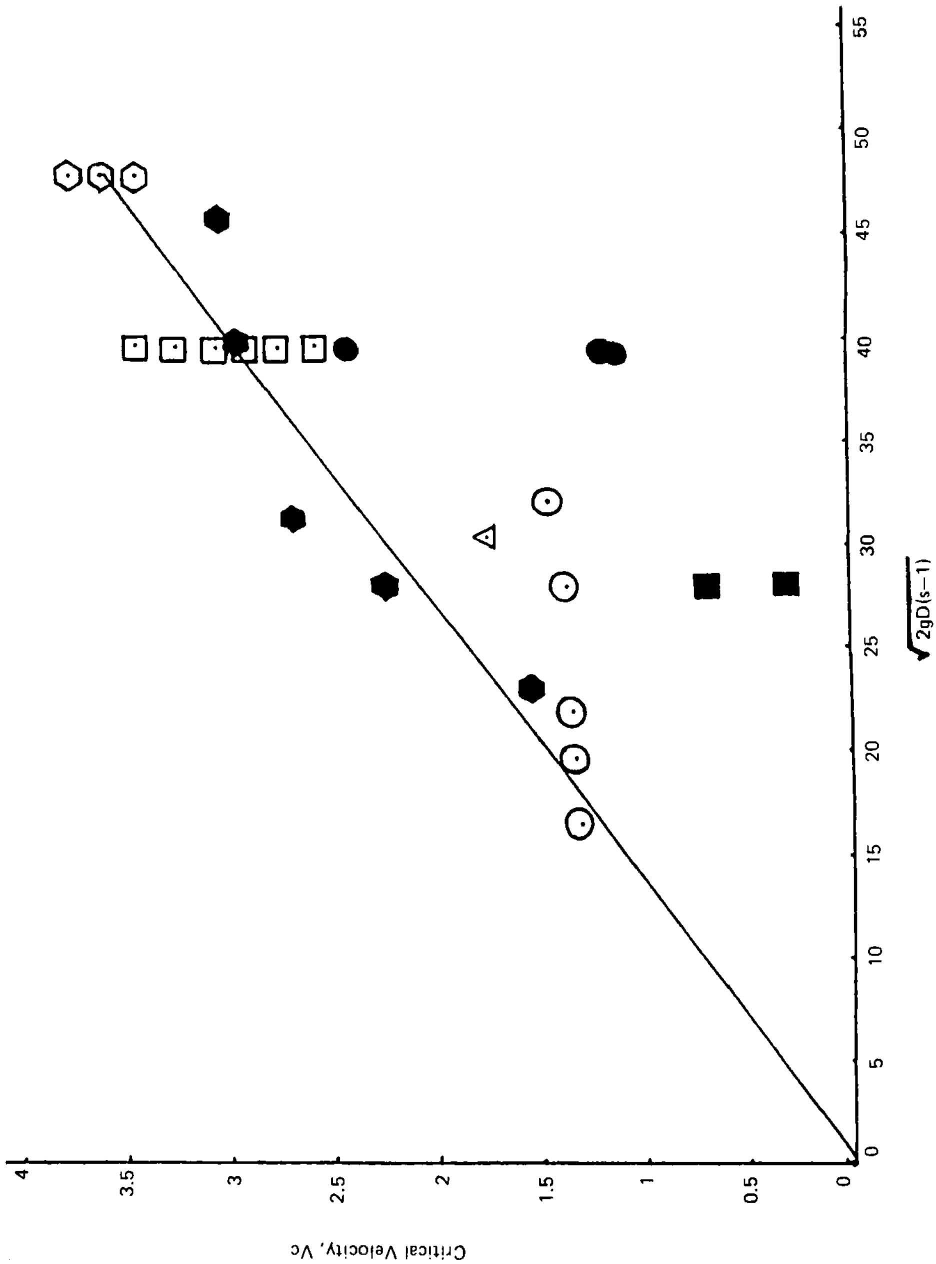

Fig. 7 The Durand plot showing the relationship between $\sqrt{2gD(s-1)}$ and the critical velocity (Vc).

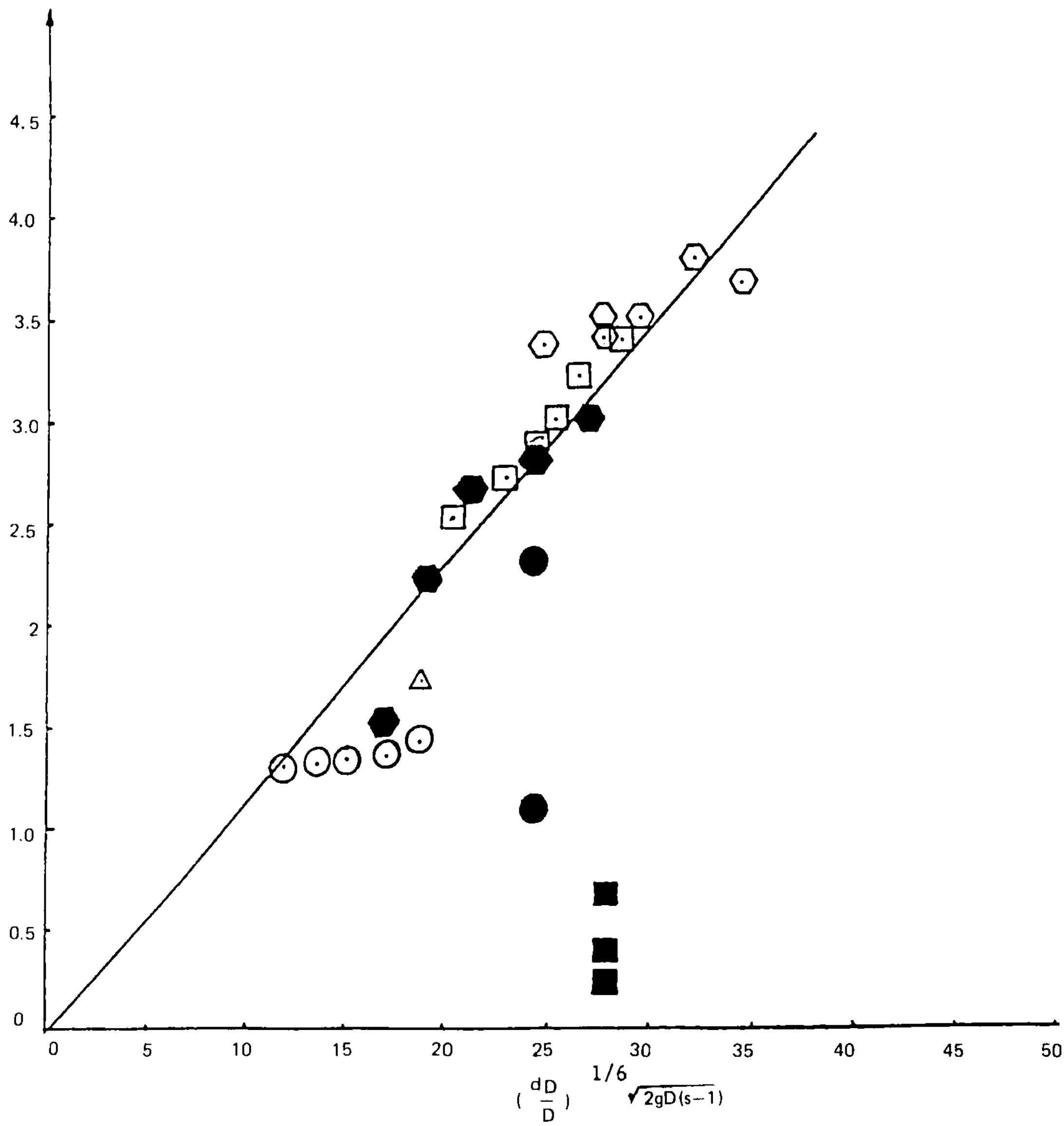

Fig. 8 The Wasp plot showing the relationship between $(\frac{dp}{D})^{1/6}\sqrt{2gD(s-1)}$ and the critical velocity (Vc).

Table 1 - Comparison of Present Work with Previous Published Correlations.

Equation Number	Correlation Name	Previous Work	Present Work	Correlation Coefficient
(5) (8)	Wicks's	$\psi=0.1\ S^3,\quad S<40$ or: $Vc=0.1\ g\ D(s-1)\ \frac{\rho dp}{\mu}$	$\psi=0.0079\ S^{2.75}$ or: $Vc=0.0209\ g^{0.8}D^{.733}(s-1)^{0.8}\frac{\rho^{0.6}}{\mu^{0.6}}d_p^{0.667}$	0.95
(1)	Durand's	$Vc=F_L\sqrt{2gD(s-1)}$	$Vc=0.0716\sqrt{2gD(s-1)}\ (F_L=.0716)$	0.70
(4)	Wasp's	$Vc=F_L'\sqrt{2gD(s-1)}\,(\frac{dp}{D})^{1/6}$	$Vc=0.1156\sqrt{2gD(s-1)}\,(\frac{dp}{D})^{1/6}\ (F_L'=0.1156)$	0.69

the viscosity factor, according to Fig. 4, is an important parameter on which Vc depends to a large extent and is ignored in the models of Durand and Wasp.

Summary and Conclusions

While the reported results pertain to an *in vitro* system several conclusions of significance to the mobility of particles in the gastrointestinal tract may be stated:-

1) For particles settling in a tube through which a fluid is flowing to move the flow rate of the fluid must exceed a certain critical value known as the critical velocity (Vc).

2) A linear relationship was found to exist between the critical velocity and the particle diameter, increasing the particle diameter results in an increase in the critical velocity.

3) A linear relationship was found to exist between the critical velocity and the density of the settled particles, increasing the density results in an increase in the critical velocity. In contrast a hyperbolic relationship between the critical velocity and the

viscosity of the flowing fluid was noted. Increasing the viscosity resulted in a decrease in the critical velocity.

4) Increasing the tube diameter from 0-0.22 cm resulted in an increase in the critical velocity, increasing the tube diameter beyond 0.22 cm however, resulted in no change in the critical velocity.

5) The data obtained in this work were found to fit the models of Wicks, Durand and Wasp for the flow conditions of settled beds.

6) It should be noted that the model used in this study is a simplified one which takes into consideration certain complications of gastrointestinal tract. Therefore its applications would be within the conditions of the experiment. Limitations to the model have been discussed elsewhere (3).

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APPENDIX A

Dimensional Analysis of Durand Correlation

The important factors controlling the slurry flow according to Durand assumptions are particle diameter, tube diameter, fluid density, solid density, and gravity forces exerted on particles, Hence one can write:

$$Vc = f(g\ \Delta\rho,\ dp,\ D,\ \rho) \qquad \text{(A-1)}$$

These five variables according to Buckingham of dimensional analysis ((16)) can be grouped in as follows:

Taking π as a product of the above variables, each raised to an unknown power, Equation (A-1) becomes as:

$$\pi = (g\Delta\rho)^a\ (dp)^b\ (D)^c\ (\rho)^d \qquad \text{(A-2)}$$

Taking mass, M, length, L, and time, t, as the basic dimensions, Equation (A-2) becomes as:

$$\pi = L\ t^{-1} = (ML^{-2}t^{-2})^a\ (L)^b\ (L)^c\ (ML^{-3})^d \qquad \text{(A-3)}$$

By comparing units on both sides of Equation (A-3) one obtains

$$L : 1 = 2a + b + c - 3d$$

$$M : 0 = a + d$$

$$t : -1 = -2a$$

Thus $a = \frac{1}{2}$, $d = -\frac{1}{2}$ and for $c = \frac{1}{2}$, $b = o$ and hence Equation (A-2) becomes as

$$Vc = \text{Constant}\ (g\Delta\rho)^{\frac{1}{2}}\ D^{\frac{1}{2}}\ \rho^{-\frac{1}{2}}$$

$$Vc = \text{Constant}\ (gD\ \frac{\Delta\rho}{\rho})^{\frac{1}{2}} \qquad \text{(A-4)}$$

and since the function under the square root is still undetermined, it can be multiplied by 2 to be consistent with the experimental results. Hence Equation (A-4) becomes:

$$Vc = F_L \sqrt{2gD(s-1)} \quad \text{(A-5)}$$

where F_L = Durand's constant

$s = \rho_s/\rho$

and $\Delta\rho = \rho_s - \rho$

APPENDIX B

Dimensional Analysis of Wasp Correlation

According to Wasp's assumptions the dimensional equation is

$$Vc = f(g\Delta\rho,\ dp,\ D,\ \rho) \quad \text{(B-1)}$$

In form of product functions Equation (B-1) becomes:

$$Vc = (g\Delta\rho)^a\ (dp)^b\ (D)^c\ (\rho)^d \quad \text{(B-2)}$$

Substituting the dimensions of different variables into Equation (B-2) one obtains:

$$\pi = L\ t^{-1} = (ML^{-2}t^{-2})^a\ (L)^b\ (L)^c\ (ML^{-3})^d \quad \text{(B-3)}$$

Comparison of units on both sides of Equation (B-3) yields

$L : 1 = 2a + b + c - 3d$

$M : 0 = a + d$

$t : -1 = -2a$

Thus $a = \frac{1}{2}$, $d = \frac{1}{2}$

For $c = \frac{1}{3}$, $b = \frac{1}{6}$

Thus Equation (B-2) becomes:

$$Vc = \text{constant}\ (g\Delta\rho)^{\frac{1}{2}}\ (dp)^{1/6}\ (D)^{1/3}\ \rho^{-\frac{1}{2}} \quad \text{(B-4)}$$

or:

$$Vc = F_L'\ \sqrt{2gD(s-1)}\ \left(\frac{dp}{D}\right)^{1/6} \quad \text{(B-5)}$$

where:

F_L' = Wasp's constant

$s = \rho_s/\rho$

$\Delta\rho = \rho_s - \rho$

APPENDIX C

Dimensional Analysis of Wicks Correlation

According to Wicks's assumption the following equation is written:

$$Vc = f(g\Delta\rho,\ D,\ \rho,\ \mu,\ d\rho) \qquad \text{(C-1)}$$

or in product form

$$Vc = (g\Delta\rho)^{a}\ (D)^{b}\ (\rho)^{c}\ (\mu)^{d}\ (d\rho)^{e} \qquad \text{(C-2)}$$

From Equation (C-2), π-equation can be written as

$$\pi = L\ t^{-1} = (ML^{-2}t^{-2})^{a}\ (L)^{b}\ (ML^{-3})^{c}\ (ML^{-1}t^{-1})^{d}\ (L)^{e} \qquad \text{(C-3)}$$

Comparing units on both sides, one obtains:

$L : 1 = -2a + b - 3c - d + e$

$M : 0 = a + c + d$

$t : -1 = 2a + d$

If we let $a = 1$ we obtain $d = -1$ and $c = 0$

For $b = 1$, $e = 1$ and thus Eq. (C-2) becomes:

$$Vc = \text{constant}\ (g\Delta\rho)^{1}\ (D)^{1}\ \mu^{-1} \qquad \text{(C-4)}$$

Setting $\Delta\rho = \rho_s - \rho$,

$s = \rho_s/\rho$, multiplying by $(\frac{\rho}{\rho})$ and

rearranging Equation (C-4), yields

$$Vc = F_w\ gD(s-1)\ \frac{\rho d}{\mu} \qquad \text{(C-5)}$$

where

Fw = Wicks's constant

= 0.1 for S < 40 as in Equation (8)